JANUARY 28TH, 1840.

JOHN ANSTER, LL.D., VICE-PRESIDENT, in the Chair.

PROFESSOR JELLETT read the following Abstract of a Paper on the Equilibrium or Motion of a Molecular System.

The object of the present paper is to deduce, on the most general theory of molecular action, the equations of equilibrium or motion of a body, solid or fluid, whose several particles have been displaced from their position of equilibrium.

The action of any one particle or molecule of a body upon another will in general depend on the *state* of the two molecules, on their *primitive positions*, and on their *displaced positions*. If it be supposed that the state of a particle, that is to say, its capacity of exerting force, is not altered by the displacement of the surrounding particles, it is plain that the force developed by the displacement of two molecules will be of the form

$$f(x, y, z, x', y', z', \xi, \eta, \zeta, \xi', \eta', \zeta',),$$

where x, y, z are the co-ordinates, and ξ, η, ζ the resolved displacements of the first particle; x', y', z', ξ', η', ζ', having the same signification for the second. The hypothesis here made may be termed the *hypothesis of independent action.*

Adopting this hypothesis, and modifying the foregoing expression by the observation that no molecular force is developed by a mere *translation* of the entire system from one position to another, the value of the force will be

$$F_0 + A(\xi' - \xi) + B(\eta' - \eta) + C(\zeta' - \zeta),$$

F_0, A, B, C being of the form

$$f(x, y, z, x', y', z'),$$

or

$$f(x, y, z, \rho, \theta, \phi),$$

where ρ, θ, ϕ are the polar co-ordinates of the second particle with regard to the first.

The tendency of this force will evidently be to change the *relative* position of the two molecules. From these principles the author has deduced, by the method of Lagrange, the equations of equilibrium or motion of any body, homogeneous or heterogeneous, whose particles satisfy the hypothesis of independent action. These are, in general, partial differential equations of the second order. If the body be homogeneous, the coefficients in these equations will become constant, and the differential coefficients of the first order will disappear.

The author finds that in the case of a homogeneous solid the number of distinct coefficients which these equations contain will be fifty-four, namely, eighteen for each equation. The equations of motion are in this case of the form

$$\begin{aligned}
\frac{d^2\xi}{dt^2} = A_1 \frac{d^2\xi}{dx^2} &+ B_1 \frac{d^2\xi}{dy^2} + C_1 \frac{d^2\xi}{dz^2}, \\
&+ A_2 \frac{d^2\eta}{dx^2} + \&c. \\
&+ A_3 \frac{d^2\zeta}{dx^2} + \&c. \\
&+ 2D_1 \frac{d^2\xi}{dydz} + \&c. + 2E_1 \frac{d^2\eta}{dydz} + \&c. + 2F_1 \frac{d^2\zeta}{dydz} + \&c.
\end{aligned}$$

$$\frac{d^2\eta}{dt^2} = \&c. \quad \frac{d^2\zeta}{dt^2} = \&c.$$

The coefficients A_1, B_1, &c. being all independent, their number will plainly be as above stated.

The author has integrated these equations for the case of plane waves and rectilinear vibrations. He finds that for each direction of wave plane there are three directions of vibration. These directions are not, however, at right angles, nor are they necessarily all real.

The author has next proceeded to examine the hypothesis that the internal moments of the system may be represented by the variation of a single function V. He finds that in this case the number of constants in the equations of motion will be reduced to thirty-six. This agrees with the result obtained by Mr. Haughton.

The author has also obtained this important result:

If V be a quadratic function of the nine quantities,

$$\frac{d\xi}{dx}, \frac{d\xi}{dy}, \frac{d\xi}{dz}, \frac{d\eta}{dx}, \frac{d\eta}{dy}, \frac{d\eta}{dz}, \frac{d\zeta}{dx}, \frac{d\zeta}{dy}, \frac{d\zeta}{dz},$$

such that the internal moments of a system or body whose particles act independently may be represented by

$$\iiint \delta V dx dy dz,$$

that part of the function which involves the products,

$$\frac{d\xi}{dx} \cdot \frac{d\eta}{dy}, \frac{d\xi}{dx} \cdot \frac{d\zeta}{dy}, \text{ \&c.},$$

must be of the form

$$L \left(\frac{d\xi}{dx} \frac{d\eta}{dy} + \frac{d\xi}{dy} \frac{d\eta}{dx} \right) + M \left(\frac{d\xi}{dx} \frac{d\zeta}{dy} + \frac{d\xi}{dy} \frac{d\zeta}{dx} \right) + \text{\&c.}$$

It is plain, then, that the coefficients of the several terms in V are not independent of one another, and cannot therefore be arbitrarily assumed. In fact there are among these coefficients nine equations of condition, whose existence is a necessary consequence of the hypothesis of independent action. Now neither the function used by Professor Mac Cullagh, nor that used by Mr. Green, satisfy these conditions. The author infers, therefore, that in media such as these writers suppose the ether to be, the *state* of each particle, i. e. its absolute power of producing motion in another particle, is changed by the displacement of the surrounding particles.

The author has then proceeded to investigate the equations of motion on this more extended supposition. He finds

that in the general case the form of these equations is not altered, and that the number of the constants remains the same. But if it be supposed that the internal moments are represented by the variation of a single function V, the present supposition differs from the hypothesis of independent action, in the absence of any restriction upon the form of V, whose coefficients are now perfectly arbitrary, and may, therefore, be assumed to satisfy at pleasure any given relations.

The Rev. Samuel Haughton communicated the following Abstract of a new Method of deducing Fresnel's Laws of Wave Propagation from a mechanical Theory.

In a memoir on a classification of elastic media, presented to this Academy in January, 1849, I deduced the general equations of motion resulting from the hypothesis, that the function on which they depend is a function of the nine differential coefficients of the displacements of each molecule. In that memoir I have also shown the possibility of the laws of wave propagation being the same in media of different molecular constitutions, and have given some examples in the theories of Light. The general function V used in that paper contains *forty-five* coefficients, and may be represented thus:

$$2V = \Sigma (a_1^2) + 2\Sigma (a_1\beta_1) + 2\Sigma (a_2 a_3) + 2\Sigma (a_1\beta_2), \qquad (1)$$

adopting the notation used in the memoir. The last term of this equation consists of *eighteen* terms, while each of the others contains *nine*. Among other hypotheses made by me at the time of writing the memoir, I assumed the coefficients of this last term to be equal in pairs, so as to reduce the total number of coefficients to thirty-six. The consequences which I deduced from this hypothesis were interesting, but I did not publish them in my memoir, as I could give no satisfactory reason for the hypothesis itself. As I conceived it at the time, it was only a mathematical assumption made to simplify my equations. Some days since, my friend Professor Jellett com-